

Substitution

meta rule

Simple

$$\frac{\phi \stackrel{val}{=} \psi}{\phi[\xi/P] \stackrel{val}{=} \psi[\xi/P]}$$

Sequential

$$\frac{\phi \stackrel{val}{=} \psi}{\phi[\xi/P][\eta/Q] \stackrel{val}{=} \psi[\xi/P][\eta/Q]}$$

Simultaneous

$$\frac{\phi \stackrel{val}{=} \psi}{\phi[\xi/P, \eta/Q] \stackrel{val}{=} \psi[\xi/P, \eta/Q]}$$

EVERY
occurrence of P
is substituted!

The rule of Leibniz

Leibniz

$$\phi \stackrel{val}{=} \psi$$

$$C[\phi] \stackrel{val}{=} C[\psi]$$

meta rule

formula that has
 ϕ as a sub formula

single
occurrence is
replaced!

Strengthening and weakening

We had

Definition: Two abstract propositions P and Q are equivalent, notation $P \stackrel{\text{val}}{=} Q$, iff

- (1) Always when P has truth value 1, also Q has truth value 1, and
- (2) Always when Q has truth value 1, also P has truth value 1.

if we relax this,
we get
strengthening

Strengthening

Definition: The abstract proposition P is stronger than Q , notation $P \stackrel{\text{val}}{=} Q$, iff

- ~~(1) Always when P has truth value 1, also Q has truth value 1, and~~
- ~~(2) Always when Q has truth value 1, also P has truth value 1.~~

Q is weaker than P

Strengthening

Definition: The abstract proposition P is stronger than Q , notation $P \models^{\text{val}} Q$, iff
always when P has truth value 1,
also Q has truth value 1.

always when P is true,
 Q is also true

Q is weaker
than P

Properties

Lemma E1: $P \stackrel{val}{=} Q$ iff $P \Leftrightarrow Q$ is a tautology.

Lemma EW1: $P \stackrel{val}{=} Q$ iff $P \stackrel{val}{\models} Q$ and $Q \stackrel{val}{\models} P$.

Lemma W2: $P \stackrel{val}{\models} P$

Lemma W3: If $P \stackrel{val}{\models} Q$ and $Q \stackrel{val}{\models} R$ then $P \stackrel{val}{\models} R$

Lemma W4: $P \stackrel{val}{\models} Q$ iff $P \Rightarrow Q$ is a tautology.

Standard Weakenings

and-or-weakening

$$P \wedge Q \stackrel{val}{\models} P$$

$$P \stackrel{val}{\models} P \vee Q$$

Extremes

$$F \stackrel{val}{\models} P$$

$$P \stackrel{val}{\models} T$$

Calculating with weakenings

(the use of standard weakenings)

Substitution

just holds

Simple

$$\frac{\text{val} \quad \phi \models \psi}{\text{val} \quad \phi[\xi/P] \models \psi[\xi/P]}$$

Sequential

$$\frac{\text{val} \quad \phi \models \psi}{\text{val} \quad \phi[\xi/P][\eta/Q] \models \psi[\xi/P][\eta/Q]}$$

Simultaneous

$$\frac{\text{val} \quad \phi \models \psi}{\text{val} \quad \phi[\xi/P, \eta/Q] \models \psi[\xi/P, \eta/Q]}$$

EVERY
occurrence of P
is substituted!

The rule of Leibniz

Leibniz

$$\frac{\phi \stackrel{val}{=} \psi}{C[\phi] \stackrel{val}{=} C[\psi]}$$

does not hold
for weakening!

formula that has
 ϕ as a sub formula

The rule of Leibniz

does not hold
for weakening!

Leibniz

$$\frac{\text{val} \quad \phi \models \psi}{\text{val} \quad C[\phi] \models C[\psi]}$$

Monotonicity

$$\frac{\text{val} \quad P \models Q}{\text{val} \quad P \wedge R \models Q \wedge R}$$

$$\frac{\text{val} \quad P \models Q}{\text{val} \quad P \vee R \models Q \vee R}$$