Model Checking of Fault-Tolerant Distributed Algorithms

Part I: Fault-Tolerant Distributed Algorithms

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Rigorous Systems Engineering

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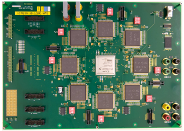
Distributed Systems



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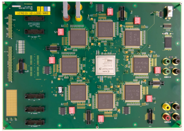


Are they always working?

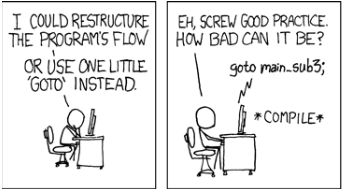
No. . . some failing systems

- Therac-25 (1985)
  - radiation therapy machine
  - gave massive overdoses, e.g., due to race conditions of concurrent tasks
- Quantas Airbus in-flight Learmonth upset (2008)
  - 1 out of 3 replicated components failed
  - computer initiated dangerous altitude drop
- Ariane 501 maiden flight (1996)
  - primary/backup, i.e., 2 replicated computers
  - both run into the same variable overflow
- Netflix outages due to Amazon's cloud (ongoing)
  - one is not sure what is going on there
  - hundreds of computers involved

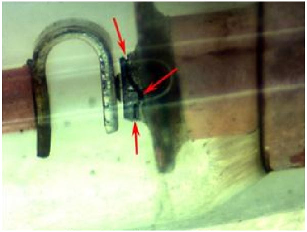
Why do they fail?

Why do they fail?

- faults at design/implementation phase



- faults at runtime
  - outside of control of designer/developer
  - e.g., to the right: crack in a diode in the data link interface of the Space Shuttle
    $\Rightarrow$ led to erroneous messages being sent



Driscoll (Honeywell)

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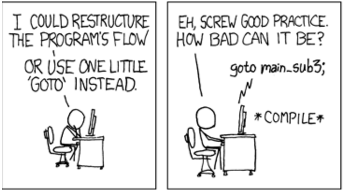
  approach:
  find and fix faults before operation
  ⇒ model checking



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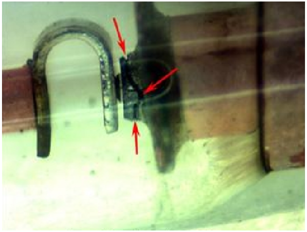

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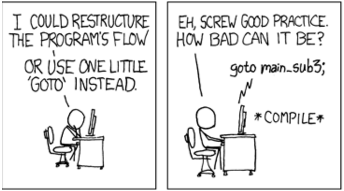

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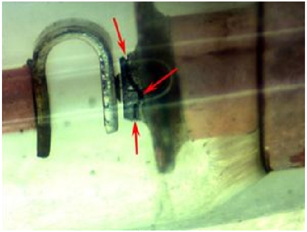
  approach:
  keep system operational despite faults
  $\Rightarrow$ fault-tolerant distributed algorithms



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Bringing both together

Goal: automatically verified fault-tolerant distributed algorithms

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model checking FTDAs is a research challenge:

- computers run independently at different speeds
- exchange messages with uncertain delays
- faults
- parameterization

. . . fault-tolerance makes model checking harder

Lecture overview

Part I: Fault-tolerant distributed algorithms

- introduction to distributed algorithms
- details of our case study algorithm
- motivation why model checking is cool

Part II: Modeling fault-tolerant distributed algorithms

- model checking challenges in distributed algorithms
- Promela, control flow automata, etc.
- model checking of small instances with Spin

Part III: Parameterized model checking of FTDAs by abstraction

- parametric interval abstraction (PIA)
- PIA data and counter abstraction
- counterexample-guided abstraction refinement (CEGAR)

Part I: Fault-Tolerant Distributed Algorithms

Distributed Systems are everywhere

What they allow to do

- share resources
- communicate
- increase performance
  - speed
  - fault tolerance

Difference to centralized systems

- independent activities (concurrency)
- components do not have access to the global state (only "local view")

Application areas

buzzwords from the 60ies

- operating systems
- (distributed) data base systems
- communication networks
- multiprocessor architectures
- control systems

New buzzwords

- cloud computing
- social networks
- multi core
- cyber-physical systems

Major challenge

Uncertainty

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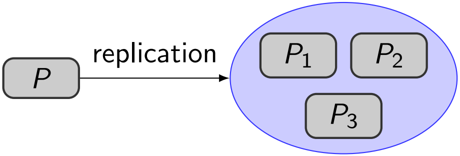
challenge in proving them correct

- large degree of non-determinism
  $\Rightarrow$ large execution and state space

From dependability to a distributed system

- Process P provides a service. We want to access it reliably
  but P may fail

From dependability to a distributed system



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- canonical approach: replication, i.e., several copies of P
  Due to non-determinism, the behavior of the copies might deviate
  (e.g. in a replicated database, transactions are committed in different orders at different sites)

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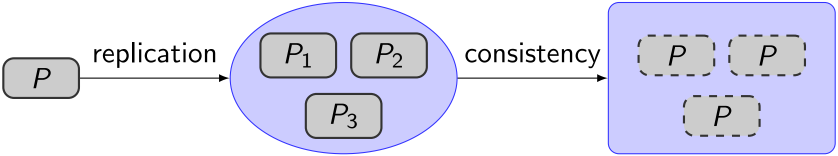

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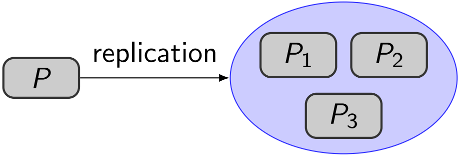
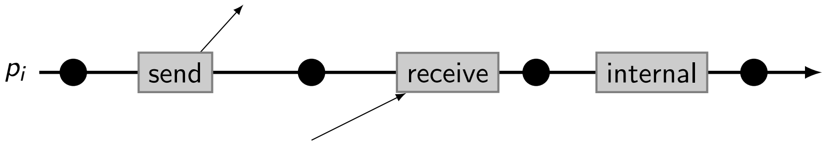
  Due to non-determinism, the behavior of the copies might deviate
  (e.g. in a replicated database, transactions are committed in different orders at different sites)
- $\Rightarrow$ we have to enforce that the copies "behave as one".
  $\Rightarrow$ Consistency in a distributed system: what does it mean to *behave as one*.

Replication — distributed systems



Distributed message passing system

multiple distributed processes p_i



- dots represent states
- a step of a process can be
  - a send step (a process sends messages to other processes)
  - a receive step (a process receives a subset of messages sent to it)
  - an internal step (a local computation)
- steps are the atomic (indivisible) units of computations

Types of Distributed Algorithms: Synchronous vs. Asynchronous

Synchronous

- all processes move in lock-step
- rounds
- a message sent in a round is received in the same round
- idealized view
- impossible or expensive to implement

Asynchronous

- only one process moves at a time
- arbitrary interleavings of steps
- a message sent is received eventually

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We focus on asynchronous algorithms here...

Asynchronous system

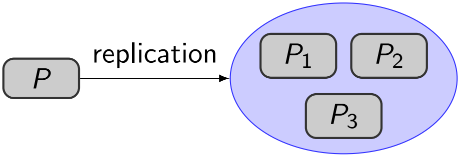
has very mild restrictions on the environment

- interleaving semantics
- unbounded message delays

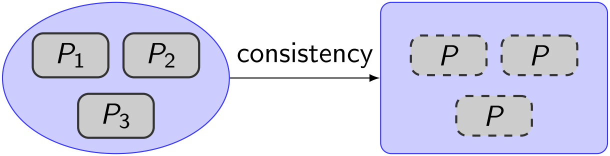
very little can be done. . .

- there is no distributed algorithm that solves consensus in the presence of one faulty process
  (as we will see, consensus is the paradigm of consistency)
- folklore explanation:
  "you cannot distinguish a slow process from a crashed one"
- real explanation:
  see intricate proof by Fischer, Lynch, and Paterson (JACM 1985)

Where we stand



What we still need...



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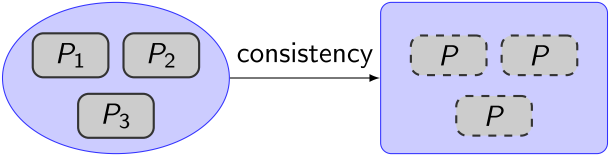

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  - atomic broadcast
  - Byzantine Generals problem
  - Byzantine agreement
  - atomic commitment
- definitions are similar but may have subtle differences

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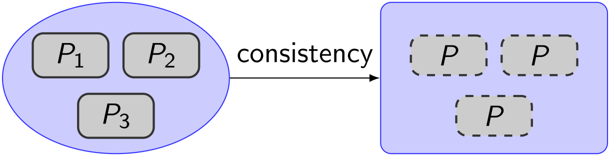

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- definitions are similar but may have subtle differences
- We use the famous Byzantine Generals to introduce this problem domain...

Fault tolerance – The Byzantine generals problem

Wiktionary:
Byzantine: adj. of a devious, usually stealthy manner, of practice.

Fault tolerance – The Byzantine generals problem

Lamport (this year's Turing laureate), Shostak, and Pease wrote in their *Dijkstra Prize in Distributed Computing* winning paper (Lamport *et al.*, 1982):

> [...] *several divisions of the Byzantine army are camped outside an enemy city, each division commanded by its own general.* [...] *However, some of the generals may be traitors* [...]

- if the divisions of loyal generals attack together, the city falls
- if only some loyal generals attack, their armies fall
- generals communicate by obedient messengers

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- the loyal generals have to agree on whether to attack.
- if all want to attack they must attack, if no-one wants to attack they must not attack

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metaphor for a distributed system where correct processes (loyal generals) act as one in the presence of faulty processes (traitors)

Byzantine generals problem cont.

In the absence of faults it is trivial to solve:

- send proposed plan ("attack" or "not attack") to all
- wait until received messages from everyone
- if a process proposed "attack" decide to attack
- otherwise, decide to not attack

Byzantine generals problem cont.

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- if a process may crash, some processes may not receive messages from everyone (but some may)
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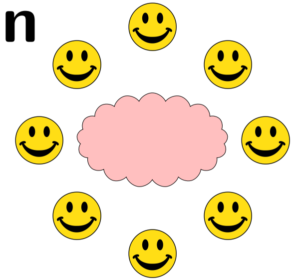
  "A tells B that C told A that C wants to attack, while C tells B that C does not want to attack" Who is lying to whom?

Fault-tolerant distributed algorithms



- n processes communicate by messages (reliable communication)
- all processes know that at most t of them might be faulty
- f are actually faulty
- resilience conditions, e.g., $n > 3t \wedge t \geq f \geq 0$
- no masquerading: the processes know the origin of incoming messages

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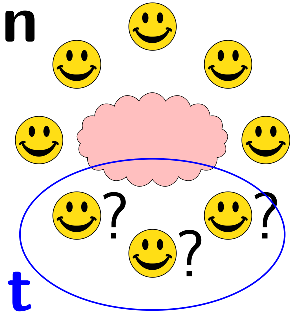

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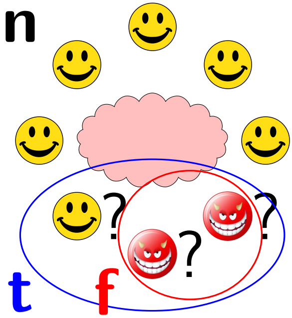

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Fault models — abstractions of reality

- clean crashes: least severe
  faulty processes prematurely halt after/before "send to all"
- crash faults:
  faulty processes prematurely halt (also) in the middle of "send to all"
- omission faults:
  faulty processes follow the algorithm, but some messages sent by them might be lost
- symmetric faults:
  faulty processes send arbitrarily to all or nobody
- Byzantine faults: most severe
  faulty processes can do anything
  encompass all behaviors of above models

Fault models — the ugly truth

A photo of a Byzantine fault:

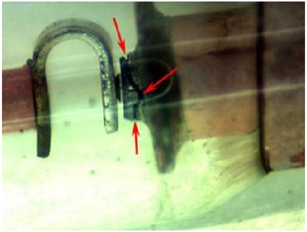

photo by Driscoll (Honeywell)
he reports Byzantine behavior on the Space Shuttle computer network

other sources of faults: bit-flips in memory, power outage, disconnection from the network, etc.

Model vs. reality: impossibilities

Hence, we would like the weakest assumptions possible. But there are theoretical limits on how weak assumptions can be made:

- consensus is impossible in asynchronous systems if there may be a crash fault, i.e., $t = 1$ (Fischer *et al.*, 1985)
- consensus is possible in synchronous systems in the presence of Byzantine faults iff $n > 3t$ (Lamport *et al.*, 1982)
- consensus is impossible in (synchronous) round-based systems if $\lfloor n/2 \rfloor$ messages can be lost per round (Santoro & Widmayer, 1989)
- fast Byzantine consensus is solvable iff $n > 5t$ (Martin & Alvisi, 2006)
- 32 different "degrees of synchrony" and whether consensus can be solved in the presence of how many faults investigated in (Dolev *et al.*, 1987)

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arithmetic resilience conditions play crucial role!

After this excursion to faults, let's go back to the problem of defining consistency

(asynchronous systems)

Defining consistency — e.g., binary consensus

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Interplay of safety and liveness makes the problem hard. . .

What if only two properties have to be satisfied?

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Give an algorithm that solves validity and termination!

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Solution: decide my own proposed value. (no need to agree)

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Solution: decide 0. (no relation to initial values required)

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Solution: do nothing (doing nothing is always safe)

Wrap-up: Intro to FTDAs

- distributed systems
- replication and consistency
- synchronous vs. asynchronous
- fault models
- example for an agreement problem: Byzantine Generals

Our case study. . .

Asynchronous FTDAs

In this lecture we consider methods for asynchronous FTDAs that either

- solve problems that are less hard than consensus:

  reliable broadcast. termination required only for specific initial state (Srikanth & Toueg, 1987). [Verified in Parts II, III]

  condition-based consensus properties required only in runs from specific initial states (Mostéfaoui *et al.*, 2003) [Verified in Part II]

  The Paxos idea fault-tolerant distributed algorithms that are safe and make progress only if you are "lucky" (Lamport, 1998) [Serious challenge]

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We use the algorithm from (Srikanth & Toueg, 1987) as running example

Asynchronous Reliable Broadcast (Srikanth & Toueg, 87)

The core of the classic broadcast algorithm from the DA literature.

```
Variables of process i
 v_i : {0, 1} initially 0 or 1
 accept_i : {0, 1} initially 0

An atomic step:
 if v_i = 1
 then send (echo) to all;

 if received (echo) from at least
   t + 1 distinct processes
   and not sent (echo) before
 then send (echo) to all;

 if received (echo) from at least
   n - t distinct processes
 then accept_i := 1;
```

Assumptions from (Srikanth & Toueg, 87)

- asynchronous interleaving
- reliable message passing (no bounds on message delays)
- at most t Byzantine faults
- resilience condition: $n > 3t \ \wedge \ t \geq f$

The spec of our case-study

Unforgeability. If $v_i = \text{FALSE}$ for all correct processes i, then for all correct processes j, accept_j remains FALSE forever.

Completeness. If $v_i = \text{TRUE}$ for all correct processes i, then there is a correct process j that eventually sets accept_j to TRUE.

Relay. If a correct process i sets accept_i to TRUE, then eventually all correct processes j set accept_j to TRUE.

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Completeness. If $v_i = \text{TRUE}$ for all correct processes i, then there is a correct process j that eventually sets accept_j to TRUE.

If all loyal generals want to attack, there shall be an attack.

Relay. If a correct process i sets accept_i to TRUE, then eventually all correct processes j set accept_j to TRUE.

If one loyal general attacks, then all loyal generals should attack.

These are the specs as given in literature: they can be formalized in LTL

Reliable broadcast vs. Consensus

Reliable broadcast: Completeness. If $v_i = \text{TRUE}$ for all correct processes i, then there is a correct process j that eventually sets accept_j to TRUE.

Consensus: Termination. Every correct process eventually decides.

Difference:

- Completeness requires to "do something" only if $\forall i.\ v_i = \text{TRUE}$, i.e., only for one specific initial state
- Termination requires to "do something" in all runs (from all initial states)
- weakening of spec makes reliable broadcast solvable in async, while consensus is not solvable

Asynchronous Reliable Broadcast (Srikanth & Toueg, 87)

The core of the classic broadcast algorithm from the DA literature.

```
Variables of process i
 v_i: {0, 1} initially 0 or 1
 accept_i: {0, 1} initially 0

An atomic step:
 if v_i = 1
 then send (echo) to all;

 if received (echo) from at least
   t + 1 distinct processes
   and not sent (echo) before
 then send (echo) to all;

 if received (echo) from at least
   n - t distinct processes
 then accept_i := 1;
```

Threshold-Guarded Distributed Algorithms

Standard construct: quantified guards (t=f=0)

- Existential Guard

  `if received` m `from` *`some`* `process then ...`

- Universal Guard

  `if received` m `from` *`all`* `processes then ...`

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what if faults might occur? 

Threshold-Guarded Distributed Algorithms

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  `if received` m `from` *`some`* `process then ...`
- Universal Guard

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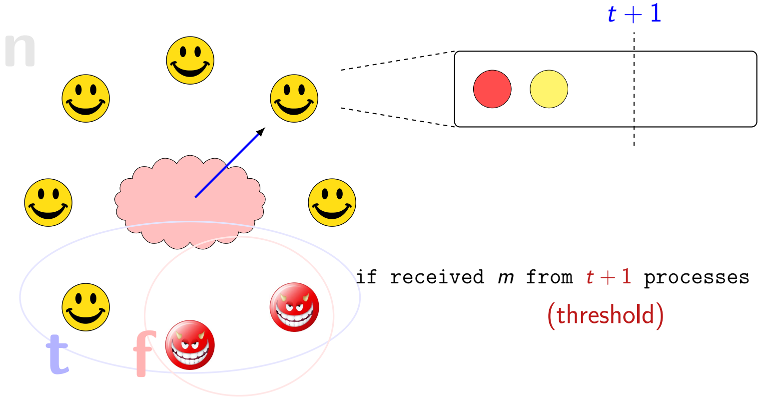
what if faults might occur?

Fault-Tolerant Algorithms: n **processes, at most** t **are Byzantine**

- Threshold Guard

  `if received` m `from` $n - t$ `processes then ...`
- (the processes cannot refer to **f**!)

Basic mechanisms used by the algorithm: thresholds

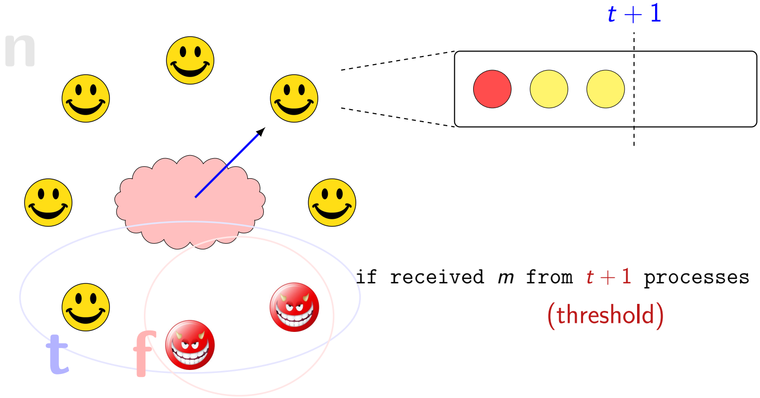


`if received` m `from` $t+1$ `processes then ...`

(threshold)

Correct processes count distinct incoming messages

Basic mechanisms used by the algorithm: thresholds



`if received` m `from` $t+1$ `processes then ...`

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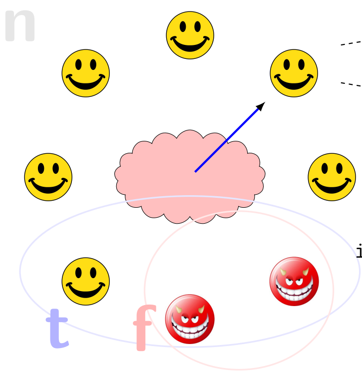
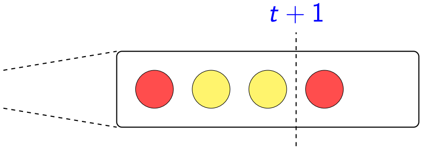
Basic mechanisms used by the algorithm: thresholds





at least one non-faulty sent the message

`if received` m `from` $t+1$ `processes then ...`

(threshold)

Correct processes count distinct incoming messages

Classic correctness argument — hand-written proofs

Proof: Unforgeability

If $v_i = \text{FALSE}$ for all correct processes i, then for all correct processes j, accept_j remains FALSE forever.

```
Variables of process i
 v_i: {0, 1} initially 0 or 1
 accept_i: {0, 1} initially 0

An atomic step:
 if v_i = 1
 then send (echo) to all;

 if received (echo) from at least
    t + 1 distinct processes
    and not sent (echo) before
 then send (echo) to all;

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 then accept_i := 1;
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- By contradiction assume a correct process sets $\text{accept}_j = 1$

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15     n - t distinct processes
16   then accept_i := 1;
```

- By contradiction assume a correct process sets $\text{accept}_j = 1$
- Thus it has executed line 16

Proof: Unforgeability

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```
1  Variables of process i
2   v_i: {0, 1} initially 0 or 1
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- By contradiction assume a correct process sets $\mathit{accept}_j = 1$
- Thus it has executed line 16
- Thus it has received $n - t$ messages by distinct processes

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```
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```

- By contradiction assume a correct process sets $\text{accept}_j = 1$
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- Thus it has received $n - t$ messages by distinct processes
- That means messages by at $n - 2t$ correct processes

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- Let p be the first correct processes that has sent (echo)

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- It did not send in line 7, as $v_p = 0$ by assumption

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2    vi: {0, 1} initially 0 or 1
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7    then send (echo) to all;
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- Thus, p sent in line 12

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- It did not send in line 7, as $v_p = 0$ by assumption
- Thus, p sent in line 12
- Based on $t + 1$ messages, i.e., 1 sent by a correct processes

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16   then accept_i := 1;
```

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- Thus it has executed line 16
- Thus it has received $n - t$ messages by distinct processes
- That means messages by at $n - 2t$ correct processes
- Let p be the first correct processes that has sent (echo)
- It did not send in line 7, as $v_p = 0$ by assumption
- Thus, p sent in line 12
- Based on $t + 1$ messages, i.e., 1 sent by a correct processes
- contradiction to p being the first one.

Proof: Completeness

If $v_i = \text{TRUE}$ for all correct processes i, then there is a correct process j that eventually sets accept_j to TRUE.

```
Variables of process i
 v_i: {0, 1} initially 0 or 1
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```

- all, i.e., at least $n - t$ correct processes execute line 7

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```

- all, i.e., at least $n - t$ correct processes execute line 7
- by reliable communication all correct processes receive all messages sent by correct processes

Proof: Completeness

If $v_i = \text{TRUE}$ for all correct processes i, then there is a correct process j that eventually sets accept_j to TRUE.

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1   Variables of process i
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- Thus, a correct process receives $n - t$ (echo) messages

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```

- all, i.e., at least $n - t$ correct processes execute line 7
- by reliable communication all correct processes receive all messages sent by correct processes
- Thus, a correct process receives $n - t$ (echo) messages
- Thus, a correct process executes line 16

Proof: Relay

If a correct process i sets accept_i to TRUE, then eventually all correct processes j set accept_j to TRUE.

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Proof: Relay

If a correct process i sets accept$_i$ to TRUE, then eventually all correct processes j set accept$_j$ to TRUE.

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1   Variables of process i
2    v_i: {0, 1} initially 0 or 1
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12   then send (echo) to all;
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14   if received (echo) from at least
15      n - t distinct processes
16   then accept_i := 1;
```

- Correct process executes line 16

Proof: Relay

If a correct process i sets accept$_i$ to TRUE, then eventually all correct processes j set accept$_j$ to TRUE.

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- Correct process executes line 16
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- Correct process executes line 16
- Thus it has received $n - t$ messages by distinct processes
- That means messages by $n - 2t$ correct processes

Proof: Relay

If a correct process i sets $accept_i$ to TRUE, then eventually all correct processes j set $accept_j$ to TRUE.

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```

- Correct process executes line 16
- Thus it has received $n - t$ messages by distinct processes
- That means messages by $n - 2t$ correct processes
- By the resilience condition $n > 3t$, we have $n - 2t \geq t + 1$

Proof: Relay

If a correct process i sets $accept_i$ to TRUE, then eventually all correct processes j set $accept_j$ to TRUE.

```
1   Variables of process i
2    v_i: {0, 1} initially 0 or 1
3    accept_i: {0, 1} initially 0
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5   An atomic step:
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- Thus at least $t + 1$ correct processes have sent (echo)

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- Thus at least $t + 1$ correct processes have sent (echo)
- By reliable communication, these messages are received by all correct processes

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- Thus at least $t + 1$ correct processes have sent (echo)
- By reliable communication, these messages are received by all correct processes
- Thus, all correct processes send (echo) in line 12

Proof: Relay

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- Thus, all correct processes send (echo) in line 12
- There are at least $n - t$ correct

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- Thus at least $t + 1$ correct processes have sent (echo)
- By reliable communication, these messages are received by all correct processes
- Thus, all correct processes send (echo) in line 12
- There are at least $n - t$ correct
- Thus, all correct processes eventually execute line 16

Problems with hand-written proofs

- code inspection becomes confusing for larger algorithms

Bracha & Toueg's algorithm (JACM 1985)

```
msg_count: array of [types: 0..1] of integer
msg: record of type: (initial, echo, ready)
value: integer

while(there is no i such that
          msg_count(initial, i) ≥ 1 or
          msg_count(echo, i) > (n + k)/2 or
          msg_count(ready, i) ≥ k + 1)
   receive(msg)
   if it is the first message received from the sender
   with these values of msg.type, msg.from
   then msg_count(msg.type, msg.value) = msg_count(msg.type, msg.value) + 1
end
for all q, send(echo, i)

while(there is no i such that
          msg_count(echo, i) > (n + k)/2 or
          msg_count(ready, i) ≥ k + 1)
   receive(msg)
   if it is the first message received from the sender
   with these values of msg.type, msg.from
   then msg_count(msg.type, msg.value) = msg_count(msg.type, msg.value) + 1
end
for all q, send(ready, i)

while(there is no i such that
         msg_count(ready, i) ≥ 2k + 1)
   receive(msg)
   if it is the first message received from the sender
   with these values of msg.type, msg.from
   then msg_count(msg.type, msg.value) = msg_count(msg.type, msg.value) + 1
end
decide i
```

FIG. 3. An asynchronous Byzantine Agreement protocol.

Part II

Condition-based consensus (Mostéfaoui *et al.*, 2003)

Function *Consensus*(v_i)

foreach $j \in [1..n]$ **do** $V_i[j] \leftarrow \perp$ **enddo**; % Intialization%

%————————————————Phase 1————————————————

UR_Broadcast PHASE1(v_i, i);

wait until (PHASE1$(-,-)$ messages have been delivered from at least $(n-f)$ processes);

foreach $j \in [1..n]$ **do if** (PHASE1(v_j, j) has been delivered) **then** $V_i[j] \leftarrow v_j$ **endif enddo**;

$w_i \leftarrow S(V_i)$; % Estimate of the decision %

%————————————————Phase 2————————————————

UR_Broadcast PHASE2(v_i, w_i, i);

repeat **wait until** (a new PHASE2(v_j, w_j, j) message has been delivered);

  if $(V_i[j] = \perp)$ **then** $V_i[j] \leftarrow v_j$ **endif**;

  if (PHASE2$(-, w, -)$ msgs with same w delivered from a majority of proc.)

    then return(w) **endif**

until (a PHASE2$(-,-,-)$ message has been delivered from each process) **endrepeat**;

return (a deterministically chosen value of V_i)

Figure 1. A Condition-Based Message Passing Consensus Protocol ($f < n/2$)

Part II

Problems with hand-written proofs

- code inspection becomes confusing for larger algorithms
- hidden assumptions
  - resilience condition
  - reliable communication (fairness)
  - non-masquerading
  - failure model

Problems with hand-written proofs

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- hidden assumptions
  - resilience condition
  - reliable communication (fairness)
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  - failure model
- re-using proofs if one of the ingredients changes?
- if I cannot prove it correct, that does not mean the algorithm is wrong
  ... how to come up with counterexamples?
- ultimate goal: verify the actual source code.
  ... it is not realistic that developers do mathematical proofs.

We have convinced a human, ...

... why should we convince a computer?

- it is easy to make mistakes in proofs

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...why should we convince a computer?

- it is easy to make mistakes in proofs
- it is easier to overlook mistakes in proofs
  - distributed algorithms require "non-centralized thinking" (untypical for the human mind)
  - many issues to consider at the same time (interleaving of steps, faults, timing assumptions)

We have convinced a human, ...

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  - distributed algorithms require "non-centralized thinking" (untypical for the human mind)
  - many issues to consider at the same time (interleaving of steps, faults, timing assumptions)
- people who tried to convince computers found bugs in published...
  - Byzantine agreement algorithm (Lincoln & Rushby, 1993)
  - clock synchronization algorithm (Malekpour & Siminiceanu, 2006)

End of Part I

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Model vs. reality: assumption coverage

Every assumption has a probability that it is satisfied in the actual system:

- $n > 3t$
  less likely than $n > t$
- every message sent is received within bounded time
  less likely than that it is eventually received
- processes fail by crashing
  less likely than they deviate arbitrarily from the prescribed behavior
- non-masquerading
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To use a distributed algorithm in practice:

- one must ensure that an assumption is suitable for a given system
- the probability that the system is working correctly is the probability that the assumptions hold
  (given that the distributed algorithm actually is correct)