## Formale Systeme Proseminar

Tasks for Week 8, 24.11.2016

Task 1 Show with derivations that the following formula is a tautology

$$\neg (P \Rightarrow Q) \Rightarrow \neg Q$$

Task 2 Give logical derivation of the following tautology

$$(P \land \neg Q) \Rightarrow \neg (P \Rightarrow Q)$$

Task 3 Give logical derivation of the following tautology

$$(\neg P \Rightarrow P) \Rightarrow P$$

Task 4 Give logical derivation of the following tautology.

$$((P \Rightarrow Q) \Rightarrow \neg P) \Rightarrow (P \Rightarrow \neg Q)$$

 ${\bf Task}~{\bf 5}~{\rm Give~logical~derivation~of~the~following~tautology}$ 

$$(P \Rightarrow Q) \lor P$$

**Task 6** Investigate whether the following formula is a tautology. If so, give a derivation to prove this; if not so, give a counterexample.

$$(P \Rightarrow Q) \Rightarrow (P \lor (Q \Rightarrow R))$$

Task 7 Give a proof of the following proposition with the help of case distinction

$$(x \ge 2 \lor x = -1) \Rightarrow x^3 - 3x - 2 \ge 0$$

for  $x \in \mathbb{R}$ .

Say precisely how you use the tautology

$$((P \lor Q) \land (P \Rightarrow R) \land (Q \Rightarrow R)) \Rightarrow R.$$