Formale Systeme Proseminar

Tasks for Week 15, 26.1.2017

Task 1 Construct a FA for the language

$$L = \{w \in \{a,b\}^* \mid w \text{ has at least three } a\text{'s or at least two } b\text{'s}\}.$$

Note that this language is a union of two languages.

Task 2 Construct a FA for the language

$$L = \{w_1 w_2 \in \{0,1\}^* \mid w_1 = 0^{2n}, w_2 = (100)^m, \text{ for some } n, m \in \mathbb{N}\}.$$

Note that the regular expression for L is $(00)^* \cdot (100)^*$.

Task 3 Let L be the language of all strings over $\{0,1\}$ that do not contain a pair of 1's that are separated by an odd number of symbols. Give the state diagram of a DFA with 5 states that recognises L.

Task 4 Construct a DFA for the language L^* where

$$L = 01 \cup (00)^* 11.$$

Task 5 Construct a DFA for the language $L_1 \cdot L_2$ where $L_1 = \{a,b\}^*$ and $L_2 = \{aabab\}$.

Task 6 Let L be a regular language, $L \subseteq \Sigma^*$. Show that the reversed language of L defined as

$$L^R = \{w \in \Sigma^* \mid w^R \in L\}$$

where reversed words are defined inductively by

$$\varepsilon^R = \varepsilon, (ua)^R = au^R \text{ for } a \in \Sigma, u \in \Sigma^*$$

is regular as well.

Hint: From an automaton for L, construct an automaton for L^R.

Task 7 Let $\Sigma = \{0,1\}$ and let

$$D = \{w \in \{0,1\}^* \mid \#_{01}(w) = \#_{10}(w)\}.$$

Thus $101 \in D$ because 101 contains a single 10 and a single 01, but $1010 \notin D$ because $\#_{01}(1010) = 1$ but $\#_{10}(1010) = 2$.

Show that D is a regular language.

Task 8 Give state diagrams of NFAs with the specified number of states recognising each of the following languages. In all parts the alphabet is $\{0, 1\}$ and the language is given via its regular expression.

   (a) The language 0 with two states.

   (b) The language 0^* with one state.

   (c) The language $(0 \cup 1)^*00$ with three states,

   (d) The language $1^* \cdot (001^+)^*$ with three states.

All the best for the rest of your studies!