Formale Systeme Proseminar

Tasks for Week 14, 19.1.2017

Task 1 Prove that $A \subseteq B \Rightarrow |A| \leq |B|$.
(Note: You need to construct an injection from A to B.)

Task 2 Prove by induction that if A is a finite set, i.e., $|A| = k$ for some $k \in \mathbb{N}$ then
$$|\mathcal{P}(A)| = 2^k.$$

Task 3 Prove that for any set X, $|\mathcal{P}(X)| = 2^{|X|}$, i.e., provide a bijection from $\mathcal{P}(X)$ to the set $\{0,1\}^X$ of all functions from X to $\{0,1\}$.

Task 4 Prove that $\aleph_0 \cdot 2 = \aleph_0$, i.e., prove that $\mathbb{N} \times \{0,1\}$ is a countable set (a set with cardinality equal to the cardinality of $\mathbb{N}$).

Task 5 Construct a DFA for the language

$$L = \{w \in \{0,1\}^* \mid w \text{ begins with a 1 and ends with a 0}\}.$$

Task 6 Construct a DFA for the language

$$L = \{w \in \{a,b\}^* \mid \text{ every } a \text{ in } w \text{ is preceeded and followed by a } b\}.$$

Task 7 Construct a DFA for the language

$$L = \{w \in \{0,1,2,3,4,5,6,7,8,9\}^* \mid w \text{ as a natural number is divisible by 3}\}.$$

Hint: A natural number is divisible by 3 iff the sum of its digits is divisible by 3.

Task 8 Construct a DFA for the language

$$L = \{w \in \{0,1\}^* \mid w \text{ ends with 11 or with 101}\}.$$

Remark: In case we do not manage to learn DFA in class, we (the people doing the instructions) will introduce DFA in the PS class and solve/discuss the last four tasks with you.