

# Formale Systeme Proseminar

Tasks for Week 12, 22.12.2016

**Task 1** Let  $A = \{a, b, c\}$ . How many equivalence relations are there on  $A$ ? List them all.

**Task 2** Consider the relation  $R \subseteq \mathbb{N} \times \mathbb{N}$  defined by

$$R = \{(n, n + 1) \mid n \in \mathbb{N}\}.$$

- (a) Find the relation  $R^2$ ,
- (b) Find the relation  $R^3$ ,
- (c) Can you think of a concise way to describe the reflexive and transitive closure relation  $R^*$ ?

**Task 3** Which of the following relations between  $A = \{a, b, c\}$  and  $B = \{1, 2\}$  are graphs of functions from  $A$  to  $B$ ?

- (a)  $R_1 = \{(a, 1), (b, 2)\}$ .
- (b)  $R_2 = \{(a, 1), (b, 1), (b, 2), (c, 1)\}$ .
- (c)  $R_3 = \{(a, 1), (b, 2), (a, 2)\}$ .
- (d)  $R_4 = \{(a, 1), (b, 2), (c, 1)\}$ .

Why?

**Task 4** Let  $A = \{a, b, c\}$  and  $B = \{1, 2\}$ . Give an example of a surjective function  $f: A \rightarrow B$ .

**Task 5** Give an example of an injective function  $f: \mathbb{N} \rightarrow \mathbb{N}$ .

**Task 6** Let  $X = \{1, 2, 3, 4, 5\}$  and consider the function  $c: \mathcal{P}(X) \setminus \{\emptyset\} \rightarrow X$  defined by  $c(Y) = |Y|$  for any  $Y \subseteq X$ ,  $Y \neq \emptyset$ . Show that  $c$  is surjective but not injective.

**Task 7** Prove Proposition S3 from the lectures, that is, show that if  $f: A \rightarrow B$  is a surjective function and  $B' \subseteq B$  then  $f(f^{-1}(B')) = B'$ .