Formale Systeme Proseminar

Tasks for Week 11, 15.12.2016

Task 1 Is it possible that a relation R is both

(a) symmetric and asymmetric?

(b) symmetric and antisymmetric?

Task 2 Let $A = \{1, 2, 3, 4\}$ and consider the relation

$$R = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,1), (3,4), (4,3)\}.$$

(a) Show that R is an equivalence relation.

(b) What are the equivalence classes of R?

Task 3 Consider the relation $R \subseteq \mathbb{Z} \times \mathbb{Z}$ given as

$$R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid \ (xy > 0) \text{ or } x = y = 0\}.$$

Prove that R is an equivalence and write down its equivalence classes.

Task 4 Prove that for any set X, the diagonal relation $\Delta_X = \{(x, x) \mid x \in X\}$ is an equivalence.

Task 5 For each of the following relations on $\mathbb{N}$ find out if it is a partial order, a strict order, a preorder, a total order, or an equivalence:

(a) xRy if and only if $|x - y|$ is a multiple of 3.

(b) xRy if and only if $x < 10$ and y is even.

Task 6 Let X be a set. Consider the relation R on $\mathcal{P}(X)$ defined by

$$(A, B) \in R \text{ iff } A \cap B = \emptyset.$$

Check if R is a partial order and/or an equivalence.

Task 7 Let $A = \{a, b, c, d\}$. For each of the following partitions of A write down the corresponding equivalence:

(a) $\{\{a, b\}, \{c, d\}\}$,

(b) $\{\{a\}, \{b, c, d\}\}$,

(c) $\{\{a\}, \{b\}, \{c\}, \{d\}\}$.

Task 8 Let $A = \{a, b, c\}$. How many equivalence relations are there on A? List them all.

Task 9 Consider the relation $R \subseteq \mathbb{N} \times \mathbb{N}$ defined by

$$R = \{(n, n+1) \mid n \in \mathbb{N}\}.$$

(a) Find the relation R^2,

(b) Find the relation R^3,

(c) Can you think of a concise way to describe the reflexive and transitive closure relation R^*?