Formale Systeme

Example Test 1, to be discussed on 25.11.2016 in the Q&A session

Task 1. (20 points) Prove that for any three sets A, B, and C we have

$$\text{If } A \subseteq B \text{ or } A \subseteq C, \text{ then } A \subseteq B \cup C.$$

Is the opposite statement true?

Task 2. (20 points) Check if the following propositional formula is a tautology. Prove your answer.

$$((P \Rightarrow Q) \Rightarrow \neg P \vee R) \Leftrightarrow (P \Rightarrow (\neg Q \vee \neg R)).$$

Task 3. (20 points) Prove with a calculation that the following abstract propositions are equivalent:

$$x \in A \cap (B^c \cup C^c) \quad \text{and} \quad x \in (A \cap B^c) \cup (A \cap C^c)$$

Task 4. (20 points) Is the following statement true? Give an explanation or a counter example.

$$\exists y \, [y \in D : \forall x \, [x \in D : P(x,y)]] \quad \stackrel{val}{\models} \quad \forall x \, [x \in D : \exists y \, [y \in D : P(x,y)]]$$

Task 5. (20 points) Prove that the following formula is a tautology.

$$\forall x[T(x) : D(x) \Rightarrow I(x)] \wedge \exists x[T(x) : D(x)] \Rightarrow \neg \forall x[T(x) : \neg I(x)].$$

Using this tautology, prove that the following statement is true:

If every difficult task is interesting and there exists a difficult task, then not all tasks are uninteresting.